MBC-0031011008 Seat No. _____

B. Sc. (Sem. I) Examination

November / December – 2016

Mathematics : Paper - 01 - A

(Theory) (Calculus)

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 (A) Answer the following questions in short [4 Marks]

1. If $f(x)=\sin x, x\in[0,\pi]$ then find the value of c using Rolle's Mean Value Theorem.
2. Define: Least Upper Bound
3. Define: Taylor's Theorem
4. Write the expansion of $\sin x$ in terms of x.

1 (B) Answer the following in brief [Any 1 out of 2] [2 Marks]

1. Find the value of c using Rolle's Theorem for $f(x)=x(x-2)^3$
2. Expand $\log(1+x)$ using Maclaurin's Theorem.

1 (C) Answer the following in detail [Any 1 out of 2] [3Marks]

1. Assuming the validity of the expansion expand $\log(1+\sin x)$ up to first three terms where $\frac{\pi}{2}<x<\frac{3\pi}{2}$, will the expansion exist for $x=-\frac{\pi}{2}$?
2. Separate the intervals in which the function $f(x)=x^3-6x^2+9x+1$ are Increasing or Decreasing.

1 (D) Answer the following in detail [Any 1 out of 2] [5 Marks]

1. State and Prove Lagrange's Mean Value Theorem.
2. Expand $e^{\sin^{-1}x}$ in terms of x.

2 (A) Answer the following questions in short [4 Marks]

1. $\lim_{x\to a}\frac{\log(x-a)}{\log(e^x-e^a)}$ is of which type of indeterminate form?

$$(D^3-6D^2+11D-6)y=e^{-2x}+e^{-3x}$$

2. Find : $\lim_{x\to 1}\frac{\log x}{x-1}$

3. State the order of the differential equation $\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$

4.Obtain the differential equation of the line $y=mx+c$.

2 (B) Answer the following in brief [Any 1 out of 2] [2 Marks]

1. Solve the differential equation $\frac{dy}{dx}+\frac{y}{x}=2$

2.Find :$\lim_{x\to 0}\frac{a^x-b^x}{x}$

2 (C) Answer the following in detail [Any 1 out of 2] [3 Marks]

1.Find :$\lim_{x\to 0}\frac{1-cosx^2}{x^2sinx^2}$

2.Solve :$(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2$

2 (D) Answer the following in detail [Any 1 out of 2] [5 Marks]

1.Solve :$(x^2+y^2)dx+2xydy=0$

2. If $\lim_{x\to\frac{\pi}{2}}\frac{cos^2x}{a-bcosecx}$ = 1 then find the values of a and b.

3 (A) Answer the following questions in short [4 Marks]

1. Write Bernouli's form of differential equation.
2. Write the condition to verify the exactness of the differential equation in Usual notations.

3.Solve : $y-px=p^2$

4. Write clairaut's form of differential equation.

3 (B) Answer the following in brief [Any 1 out of 2] [2 Marks]

1.Solve :$e^y\left(\frac{dy}{dx}+1\right)=e^x$

2.Solve : $p^2=(p-1)y$

3 (C) Answer the following in detail [Any 1 out of 2] [3 Marks]

1.Solve :$x^2p^2xyp-6y^2=0$

2.Solve :$x^2(y-px)=yp^2$

3 (D) Answer the following in detail [Any 1 out of 2] [5 Marks]

1.Solve : $p^2 - 6px + 3y = 0$

2.Solve : $y^2(y - xp) = x^4 p^2$

4 (A) Answer the following questions in short [4 Marks]

1. Find the complementary function of $(D^2 - 7D + 12)y = e^x$

2. Write the expansion for $(1 + D)^{-1}$

3.Evaluate : $\frac{1}{D^2 + 7} \sin 2x$

4. Evaluate: $\frac{1}{(D-1)^2} x^2$

4 (B) Answer the following in brief [Any 1 out of 2] [2 Marks]

1.Solve : $(D^4 + 4)y = 0$

2. Find $\frac{1}{D^3}(5x^2)$

4 (C) Answer the following in detail [Any 1 out of 2] [3 Marks]

1.Prove in usual notations $\frac{1}{D-a} X = e^{ax} \int X e^{-ax} dx$

2.Solve : $\frac{d^3y}{dx^3} - 12\frac{d^2y}{dx^2} + 6\frac{dy}{dx} - 8y = e^{2x}$

4 (D) Answer the following in detail [Any 1 out of 2] [5 Marks]

1.Prove in usual notations: $\frac{1}{f(D)} e^{ax} V = e^{ax} \frac{1}{f(D+a)} V$; where V function of x

2.Solve : $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$

5 (A) Answer the following questions in short [4 Marks]

1. Find C.F. for $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 2\log x$

2. Find P.I. for $x\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=\frac{1}{x}$

3. Find P.I. for $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\,l\log x}{x^2}$

4. Find C.F. for $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log x$

5 (B) Answer the following in brief [Any 1 out of 2] [2 Marks]

1.Solve : $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-9y=0$

2. Find P.I. for $(x^2D^2+xD-1)y=x^m$

5 (C) Answer the following in detail [Any 1 out of 2] [3 Marks]

1.Solve : $x^2\frac{d^2y}{dx^2}-2y=x^2+\frac{1}{x}$

2.Find C.F. for $(x^3D^3+2x^3D^2+2)y=10\left(x+\frac{1}{x}\right)$

5 (D) Answer the following in detail [Any 1 out of 2] [5 Marks]

1.Solve : $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$

2. Solve : $x^2\frac{d^2y}{dx^2}+7x\frac{dy}{dx}+5y=x^5$